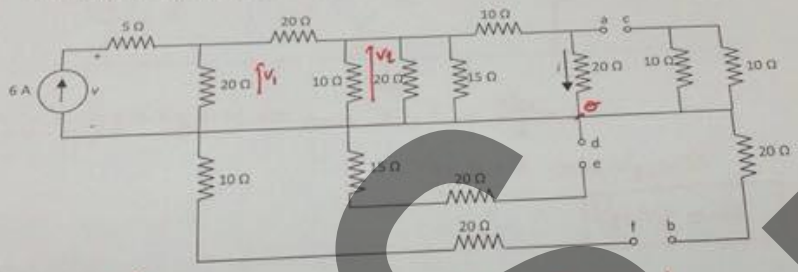
Solution

Introduction to Electrical Engineering
Summer 2014
Test 1
Duration: 60 min
Instructor: Dr. B. Habib
Observations: THE EXAM IS CLOSED-BOOK AND OVER 100.

Name :
ID :

Question 1 : Network analysis by using series and parallel equivalents. **(20 pts)**

Find the values of v (Ohm's Law), i and V_{ab}, where a, b, c, d, e and f are open-circuits.



$$Req = 5 + \Big(\Big[\underbrace{(20+10)\,//\,15\,//\,20\,//\,10}_{\frac{1}{1/15+1/20+1/10+1/30}=4} + 20\Big]\,//\,20\Big)$$

$$\underbrace{24 \;//\; 20}_{\frac{1}{1/24+1/20}=20{,}909}$$

(8 pt)

$\boxed{Req = 15{,}909}$

$\boxed{V = Req \cdot i = 15{,}909 \cdot 6A = 95{,}4545\,V}$ (4 pt)

$V_{ab} = V_{a\theta} = i \cdot 20$

$V_1 = \frac{10{,}909}{5+10{,}909} \cdot V \quad (V\text{-}D) = 65{,}4543\,V.$

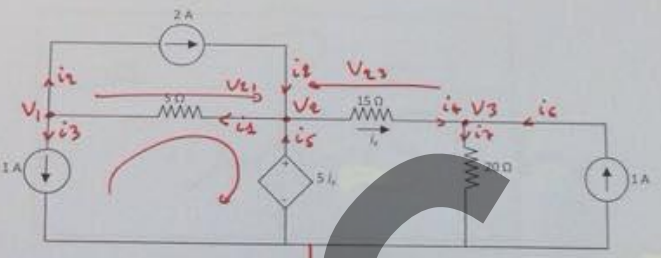
$V_2 = \frac{4}{4+20} \cdot V_1 = 10{,}909\,V.$

$V_{a\theta} = \frac{20}{10+20} \cdot V_2 = \boxed{7{,}2727V = V_{ab}}$ (4 pt)

$\Rightarrow \boxed{i = \frac{V_{ab}}{20} = 0{,}363\,A}$ (4 pt) (ohm's Law)

Question 2 : Node-voltage analysis. **(20 pts)**

Use the node-voltage technique to solve the value of i_x. Select the location of the reference node to minimize the number of unknown node-voltages.



Node 1: $i_2 + i_3 - i_1 = 0 \Rightarrow 2 + 1 - \frac{V_{21}}{5} = 0$

$-\frac{V_{21}}{5} = -3 \Rightarrow V_{21} = 15$

$\boxed{V_2 - V_1 = 15}$ (1)

Node 2:

$-i_2 + i_1 + i_4 - i_5 = 0$ where $\boxed{i_x = \frac{V_{23}}{15} = \frac{V_2 - V_3}{15}}$

$-2 + \frac{V_2 - V_1}{5} + \frac{V_2 - V_3}{15} - \frac{5(V_2 - V_3)}{15} = 0$

$-30 + 3V_2 - 3V_1 + V_2 - V_3 - 5V_2 + 5V_3 = 0$

$\boxed{-3V_1 - V_2 + 4V_3 - 30 = 0}$ (2)

Node 3:

$i_6 + i_4 - i_7 = 0$

$\frac{V_2 - V_3}{15} + 1 - \frac{V_3}{20} = 0$

$5 + \frac{V_2}{3} - \frac{V_3}{3} - \frac{V_3}{4} = 0 \Rightarrow \boxed{5 + \frac{1}{3}V_2 - 0{,}5833V_3 = 0}$ (3)

(1) + (2) + (3) $\Rightarrow$

(1') $V_1 = 13{,}76$ V

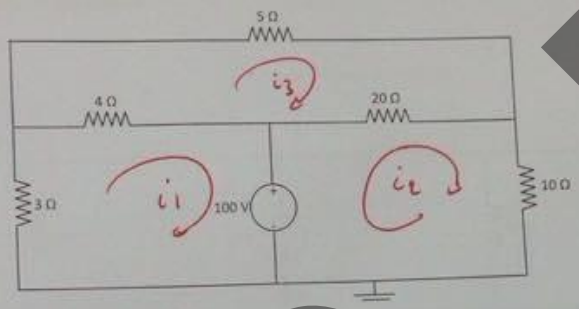
(2') $V_2 = 25{,}01$ V $\Rightarrow i_x = 0{,}25$ A

(3') $V_3 = 28{,}76$ V

10pt

Question 3 : Mesh-current analysis. **(20 pts)**

Solve the power delivered by the voltage source using the mesh-current method.



Mesh 1: $3i_1 + 4(i_1 - i_3) + 100 = 0$ (1)

Mesh 2: $-100 + (i_2 - i_3).20 + 20i_2 = 0$ (2)

Mesh 3: $5.i_3 + 4(i_3 - i_1) + 20(i_3 - i_2) = 0$ (3)

(1) + (2) + (3) → $i_1 =$ [illegible] 13,87 A.

$i_2 =$ [illegible] 3,8 A

$i_3 =$ [illegible] 0,711 A.

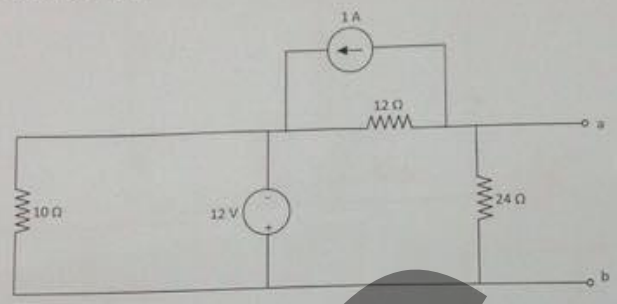
$P = V.I$

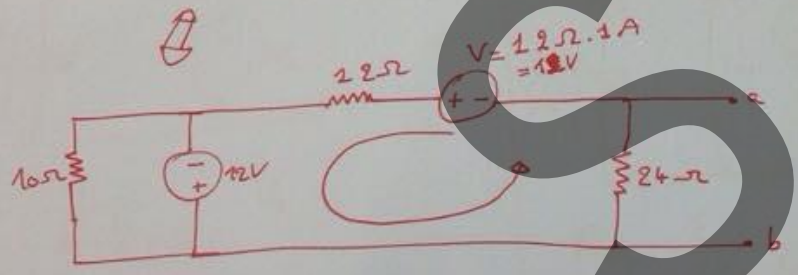
$= 100.(i_2 - i_1)$

$=$ [illegible] 1767 W

Question 4 : Thevenin and Norton equivalent circuits. **(20 pts)**

Find the Thevenin and Norton equivalent circuits. Then calculate the maximum power delivered by the circuit.





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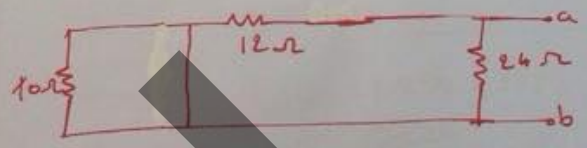
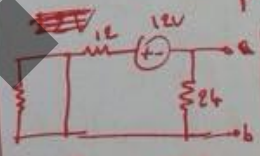
Finding R_t:



$$R_t = \frac{1}{1/12 + 1/24} = 8\,\Omega.$$

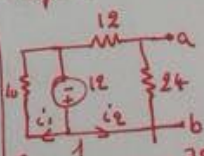
Finding $V_{oc} = V_{ab}$:

Applying the superposition principle:



$$i = \frac{12}{12+24} = 0{,}333\,A.$$

$$V_{ab} = 24 \cdot i = 8\,V.$$



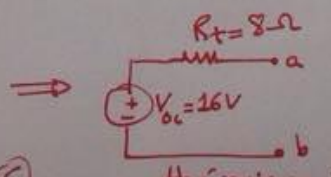
$$R = \frac{1}{1/10 + 1/12+24} = 7{,}8\,\Omega.$$

$$i = \frac{12}{7{,}8} = 1{,}53\,A$$

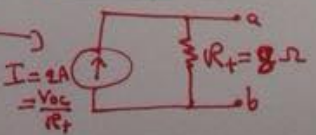
$$i_2 = 0{,}3333\,A$$

$$V_{ab} = i_2 \cdot 24 = 8\,V.$$

$$\Rightarrow V_{ab\,total} = 16\,V$$



thévenin equivalent circuit



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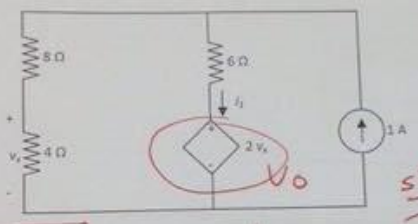
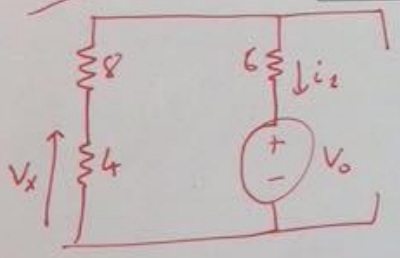
$$\Rightarrow P_{L\,max} = V_t^2 / 4R_t = 8\,W.$$

Question 5 : Superposition Principle. **(20 pts)**

Use the superposition method to solve i_1.



step1



$$V_x = \frac{4}{4+8+6} \cdot V_o$$

$$= 0,2222 \cdot V_o$$

$$i_1 = -\frac{V_o}{18}$$

$$\Rightarrow V_{x\,total} = 0,2222 + 1,3333$$

$$i_1 = 0,6666 - \frac{V_o}{18}$$

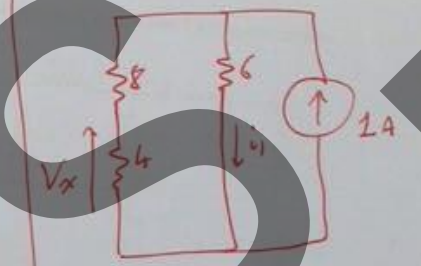
$$V_o = 2V_x \Rightarrow V_x = 0,2222 \cdot 2 \cdot V_x + 1,3333$$

$$= 0,4444 V_x + 1,3333$$

$$\Rightarrow V_x = 2,4\ V$$

$$\Rightarrow \boxed{i_1 = 0,533A.}$$

step2



$$R_{eq} = \frac{1}{\frac{1}{6} + \frac{1}{8+4}} = 4$$

$$V = R \cdot I = 4V.$$

$$\Rightarrow V_x = \frac{4}{4+8} \cdot 4V$$

$$= 1,3333\ V.$$

$$i_2 = \frac{8+4}{8+4+6} \cdot 1$$

$$= 0,6666\ A$$